Mathematical Association of America (AMC)
WYOMING SEMINARY COLLEGE PREP
AMC 12 A - Fall 2024 (11/06/2024)
Individual Student Report
Grade Report

DataMate™

Student ID	Student Name	Grade	Room/Class	Date
199153	Tsui, Andrew	11	AMC12A-2024 - 0015f00001aAnqDAAS	11/6/2024

Question	Answer/Pts Earned	Correct	Points	Max Pts
Multiple Choice				
Problem 1	A	A	6	-
Problem 2	B	B	6	-
Problem 3	B	B	6	-
Problem 4	D	D	6	-
Problem 5	D	D	6	-
Problem 6	B	B	6	-
Problem 7	D	D	6	-
Problem 8	A	A	6	-
Problem 9	E	E	6	-
Problem 10	C	C	6	-
Problem 11	E	D	0	-
Problem 12	E	E	6	-
Problem 13	D	D	6	-
Problem 14		C	1.5	-
Problem 15	D	D	6	-
Problem 16	A	C	0	-
Problem 17	D	D	6	-
Problem 18	A	A	6	-
Problem 19	D	D	6	-
Problem 20	D	D	6	-
Problem 21	B	B	6	-
Problem 22	E	C	0	-
Problem 23	A	B	0	-
Problem 24	B	D	0	-
Problem 25		B	1.5	-

Multiple Choice	
Score	111/150
Overall Summary	

199153 Tsui, Andrew
Grade: 11 Test ID: 99932 on 11/6/2024, Page: 1
Section: AMC12A-2024 - 0015f00001aAnqDAAS
WYOMING SEMINARY COLLEGE PREP

1	**A**	B	C	D	E
2	A	**B**	C	D	E
3	A	**B**	C	D	E
4	A	B	C	**D**	E
5	A	B	C	**D**	E
6	A	**B**	C	D	E
7	A	B	C	**D**	E
8	**A**	B	C	D	E
9	A	B	C	D	**E**
10	A	B	**C**	D	E
11	A	B	C	D	**E**
12	A	B	C	D	**E**
13	A	B	C	**D**	E
14	A	B	C	D	E
15	A	B	C	**D**	E
16	**A**	B	C	D	E
17	A	B	C	**D**	E
18	**A**	B	C	D	E
19	A	B	C	**D**	E
20	A	B	C	**D**	E
21	A	**B**	C	D	E
22	A	B	C	D	**E**
23	**A**	B	C	D	E
24	A	**B**	C	D	E
25	A	B	C	D	E

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MAA

AMC AMERICAN MATHEMATICS COMPETITION

MAA American Mathematics Competitions

76th Annual

AMC 12 A

Wednesday, November 6, 2024

INSTRUCTIONS

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
8. When you finish with the competition, please follow the directions of your competition manager.

The problems and solutions for this AMC 12 A were prepared by the MAA AMC 10/12 Editorial Board under the direction of Gary Gordon and Carl Yerger, co-Editors-in-Chief.

The MAA AMC office reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this AMC 12 will be invited to take the 43rd annual American Invitational Mathematics Examination (AIME) on Thursday, February 6, 2025, or Wednesday, February 12, 2025. More details about the AIME can be found at maa.org/AMC.

1. What is the value of $101 \cdot 9{,}901 - 99 \cdot 10{,}101$?

(A) 2 **(B)** 20 **(C)** 21 **(D)** 200 **(E)** 2020

2. A model used to estimate the time it will take to hike to the top of a mountain on a trail is of the form $T = aL + bG$, where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimate it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?

(A) 240 **(B)** 246 **(C)** 252 **(D)** 258 **(E)** 264

3. The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

4. What is the least value of n such that $n!$ is a multiple of 2024 ?

(A) 11 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 253

5. A data set containing 20 numbers, some of which are 6, has mean 45. When all the 6s are removed, the data set has mean 66. How many 6s were in the original data set?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

6. The product of three integers is 60. What is the least possible positive sum of the three integers?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 13

7. In $\triangle ABC$, $\angle ABC = 90^\circ$ and $BA = BC = \sqrt{2}$. Points $P_1, P_2, \ldots, P_{2024}$ lie on hypotenuse $\overline{AC}$ so that $AP_1 = P_1P_2 = P_2P_3 = \cdots = P_{2023}P_{2024} = P_{2024}C$. What is the length of the vector sum

$$\overrightarrow{BP_1} + \overrightarrow{BP_2} + \overrightarrow{BP_3} + \cdots + \overrightarrow{BP_{2024}}\ ?$$

(A) 1011 **(B)** 1012 **(C)** 2023 **(D)** 2024 **(E)** 2025

8. How many angles θ with $0 \le \theta \le 2\pi$ satisfy $\log(\sin(3\theta)) + \log(\cos(2\theta)) = 0$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M ?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

10. Let α be the radian measure of the smallest angle in a $3-4-5$ right triangle. Let β be the radian measure of the smallest angle in a $7-24-25$ right triangle. In terms of α, what is β ?

(A) $\frac{\alpha}{3}$ **(B)** $\alpha - \frac{\pi}{8}$ **(C)** $\frac{\pi}{2} - 2\alpha$ **(D)** $\frac{\alpha}{2}$ **(E)** $\pi - 4\alpha$

11. There are exactly K positive integers b with $5 \leq b \leq 2024$ such that the base-b integer 2024_b is divisible by 16 (where 16 is in base ten). What is the sum of the digits of K ?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 20 **(E)** 21

12. The first three terms of a geometric sequence are the integers a, 720, and b, where $a < 720 < b$. What is the sum of the digits of the least possible value of b ?

(A) 9 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 21

13. The graph of $y = e^{x+1} + e^{-x} - 2$ has an axis of symmetry. What is the reflection of the point $\left(-1, \frac{1}{2}\right)$ over this axis?

(A) $\left(-1, -\frac{3}{2}\right)$ **(B)** $(-1, 0)$ **(C)** $\left(-1, \frac{1}{2}\right)$ **(D)** $\left(0, \frac{1}{2}\right)$ **(E)** $\left(3, \frac{1}{2}\right)$

14. The numbers, in order, of each row and the numbers, in order, of each column of a 5×5 array of integers form an arithmetic progression of length 5. The numbers in positions $(5,5)$, $(2,4)$, $(4,3)$, and $(3,1)$ are 0, 48, 16, and 12, respectively. What number is in position $(1,2)$?

$$\begin{bmatrix} . & ? & . & . & . \\ . & . & . & 48 & . \\ 12 & . & . & . & . \\ . & . & 16 & . & . \\ . & . & . & . & 0 \end{bmatrix}$$

(A) 19 **(B)** 24 **(C)** 29 **(D)** 34 **(E)** 39

15. The roots of $x^3 + 2x^2 - x + 3$ are p, q, and r. What is the value of

$$\left(p^2 + 4\right)\left(q^2 + 4\right)\left(r^2 + 4\right)\ ?$$

(A) 64 **(B)** 75 **(C)** 100 **(D)** 125 **(E)** 144

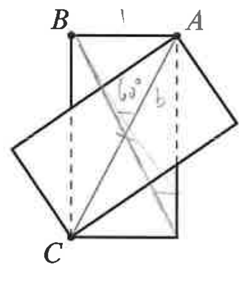
16. A set of 12 tokens—3 red, 2 white, 1 blue, and 6 black—is to be distributed at random to 3 game players, 4 tokens per player. The probability that some player gets all the red tokens, another player gets all the white tokens, and the remaining player gets the blue token can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 387 **(B)** 388 **(C)** 389 **(D)** 390 **(E)** 391

17. Integers a, b, and c satisfy $ab + c = 100$, $bc + a = 87$, and $ca + b = 60$. What is $ab + bc + ca$?

(A) 212 **(B)** 247 **(C)** 258 **(D)** 276 **(E)** 284

18. On top of a rectangular card with sides of length 1 and $2 + \sqrt{3}$, an identical card is placed so that two of their diagonals line up, as shown ($\overline{AC}$, in this case).



Continue the process, adding a third card to the second, and so on, lining up successive diagonals after rotating clockwise. In total, how many cards must be used until a vertex of a new card lands exactly on the vertex labeled B in the figure?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** No new vertex will land on B.

19. Cyclic quadrilateral $ABCD$ has lengths $BC = CD = 3$ and $DA = 5$ with $\angle CDA = 120°$. What is the length of the shorter diagonal of $ABCD$?

(A) $\frac{31}{7}$ **(B)** $\frac{33}{7}$ **(C)** 5 **(D)** $\frac{39}{7}$ **(E)** $\frac{41}{7}$

20. Points P and Q are chosen uniformly and independently at random on sides $\overline{AB}$ and $\overline{AC}$, respectively, of equilateral triangle $\triangle ABC$. Which of the following intervals contains the probability that the area of $\triangle APQ$ is less than half the area of $\triangle ABC$?

(A) $\left[\frac{3}{8}, \frac{1}{2}\right]$ **(B)** $\left(\frac{1}{2}, \frac{2}{3}\right]$ **(C)** $\left(\frac{2}{3}, \frac{3}{4}\right]$ **(D)** $\left(\frac{3}{4}, \frac{7}{8}\right]$ **(E)** $\left(\frac{7}{8}, 1\right]$

21. Suppose that $a_1 = 2$ and the sequence (a_n) satisfies the recurrence relation

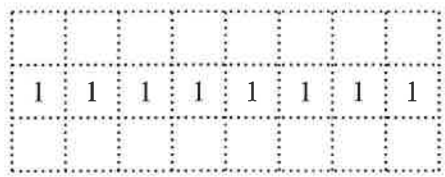
$$\frac{a_n - 1}{n - 1} = \frac{a_{n-1} + 1}{(n - 1) + 1}$$

for all $n \geq 2$. What is the greatest integer less than or equal to

$$\sum_{n=1}^{100} a_n^2 \text{ ?}$$

(A) 338,550 **(B)** 338,551 **(C)** 338,552 **(D)** 338,553 **(E)** 338,554

22. The figure below shows a dotted grid 8 cells wide and 3 cells tall consisting of $1'' \times 1''$ squares. Carl places 1-inch toothpicks along some of the sides of the squares to create a closed loop that does not intersect itself. The numbers in the cells indicate the number of sides of that square that are to be covered by toothpicks, and any number of toothpicks are allowed if no number is written. In how many ways can Carl place the toothpicks?



(A) 130 **(B)** 144 **(C)** 146 **(D)** 162 **(E)** 196

23. What is the value of

$$\tan^2\frac{\pi}{16}\cdot\tan^2\frac{3\pi}{16}+\tan^2\frac{\pi}{16}\cdot\tan^2\frac{5\pi}{16}+\tan^2\frac{3\pi}{16}\cdot\tan^2\frac{7\pi}{16}+\tan^2\frac{5\pi}{16}\cdot\tan^2\frac{7\pi}{16}?$$

(A) 28 **(B)** 68 **(C)** 70 **(D)** 72 **(E)** 84

24. A *disphenoid* is a tetrahedron whose triangular faces are congruent to one another. What is the least total surface area of a disphenoid whose faces are scalene triangles with integer side lengths?

(A) $\sqrt{3}$ **(B)** $3\sqrt{15}$ **(C)** 15 **(D)** $15\sqrt{7}$ **(E)** $24\sqrt{6}$

25. A graph is *symmetric* about a line if the graph remains unchanged after reflection in that line. For how many quadruples of integers (a, b, c, d), where $|a|, |b|, |c|, |d| \le 5$ and c and d are not both 0, is the graph of

$$y = \frac{ax+b}{cx+d}$$

symmetric about the line $y = x$?

(A) 1282 **(B)** 1292 **(C)** 1310 **(D)** 1320 **(E)** 1330

$-\frac{2}{1} = -2 = p+q+r$ $\quad -1 = pqr$

$p^2q^2r^2 + 4(pq^2 + qr^2 + pr^2) = 16(p^2+q^2+r^2) + 64$

$(p+q+r)^2 - 4 = 2(pq + qr + pr) + p^2q^2r^2$

$a^2 = 5^2 + 3^2 - 2ab\cos 120°$

$2c + 24 + 30 \cdot \frac{1}{2} = 49$ $\quad a = 7$

$a + b + c = 100$

$c = 100 - ab$

$a \cdot b = c \cdot d$

$z = 23 - 18\cos$

18

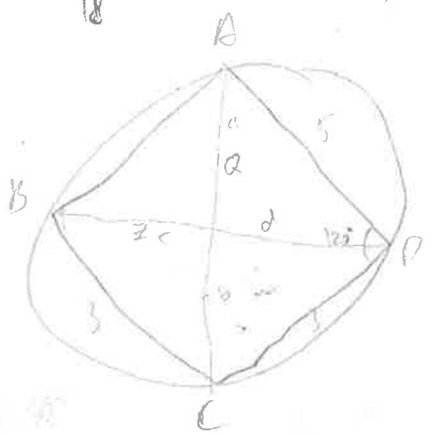
$z = \frac{39}{7}$

$b = 12$

276

$100b - ab^2 + a = 50$

$100a - a^2b + b = 50$



$27 = a - b + 100b - 100a = a(1-100) + b(100-1)$

$27 = -99a + 99b$

$> \frac{3}{4}$

$101n - 99(n+200) = ?$

$99n = 19800$

$2n - 19800 \quad n = 9901$

$19802 - 19800 = 2$

$$\begin{array}{r} 99 \\ \times 2 \\ \hline 198 \end{array} \qquad \begin{array}{r} 9901 \\ \times 2 \\ \hline 19802 \end{array}$$

$y' = e^{x+1} - e^{-x} \Rightarrow e^{x-1} = \frac{1}{e^x}$

$e^{2x-1} = 1$

$x = \frac{1}{2} \Rightarrow e^0 = 1$

ans

$T = 1.2a + 1100G = 69$

$T = 1.5a + 800G = 69$

$0.3a + 275G = \frac{69}{4}$

$a = 30 \quad G = \frac{3}{100}$

$T = 4.2a + 4000G = 126 + 120 = 246$

$$\begin{array}{r} 69 \\ \times 5 \\ \hline 345 \end{array} \qquad \begin{array}{r} 69 \\ \times 4 \\ \hline 276 \\ +345 \\ \hline 621 \end{array}$$

$b = \frac{723}{a}$

2024, 2, 1012, 2, 506, 2, 253, 23, 11

68, 2, 34, 2, 17

$4 \cdot 2^2 \cdot 3 \cdot 5$

$4(5 \cdot 20) = 400$

$\frac{400 - (6x)}{23 - x} = 66$

$1320 - 66x$

$60x = 420$

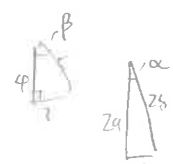
$x = 7$

$$\begin{array}{r} 48 \\ 15\overline{)720} \\ 60 \\ \hline 120 \end{array} \qquad \begin{array}{r} 48 \\ \times 16 \\ \hline 480 \\ 288 \\ \hline 768 \end{array}$$

$\log(\sin 3\theta + \cos 2\theta) = 0$

$\log(1) = 0$

$\sin 3\theta + \cos 2\theta = 1$



$a^2 = M + 1213$

$b^2 = M + 3773$

$b^2 - a^2 = 3773 - 1213 = 2560$

$(b-a)(b+a) = 2560$

$2^{9} \cdot 5$

2560, 10, 256, 2, 128, 2, 64, 2, 32, 16

$\tan^{-1}\left(\frac{3}{4}\right) = \beta \qquad \beta + 2\alpha = \frac{\pi}{2}$

$\tan^{-1}\left(\frac{7}{24}\right) = \alpha \qquad \beta = -2\alpha + \frac{\pi}{2}$

$\tan(2\alpha)$

$\tan(2\alpha) + \tan(\beta) = 1$

$2\alpha + \beta = \arccos(1) = \frac{\pi}{2}$